Ko`pburchaklar

REJA:

1. *Ko`pyoqlar*
2. *PRIZMA*
3. *Ko`pburchak ortogonll proeksiyasining юzi*
4. *PRIZMA SIRTINING ЮZI*
5. *KO`PYOQLAR HAJMLARINING UMUMIY xossalari .*
6. *TO`G`RI BURCHAKLI parallelepipedning hajmi*
7. *Eyler teoremasi*
8. *Koshi teoremasi*
9. *Koshi masalasi va uning qo`yilishida xarakteristikalarning roli.*
10. *Muntazam ko`pyoqlar*

Bir necha ko`pburchak birlaшmasidan iborat notekis figuralarga doir misollar VIII sinf kursidan ma`lum. Bunday figuralar- ga to`g`ri prizmaning yon sirti (1- rasm), piramidaning sirti (2- rasm) kiradi. Bu figuralar solda ko`p yoqli sirtlarga misol- lardir.

Чekli sondagi ko`pburchaklarning quyidagi шartlarni qanoat- lantiruvchi birlaшmasi solda ko`p yoqli sirt deyiladi:

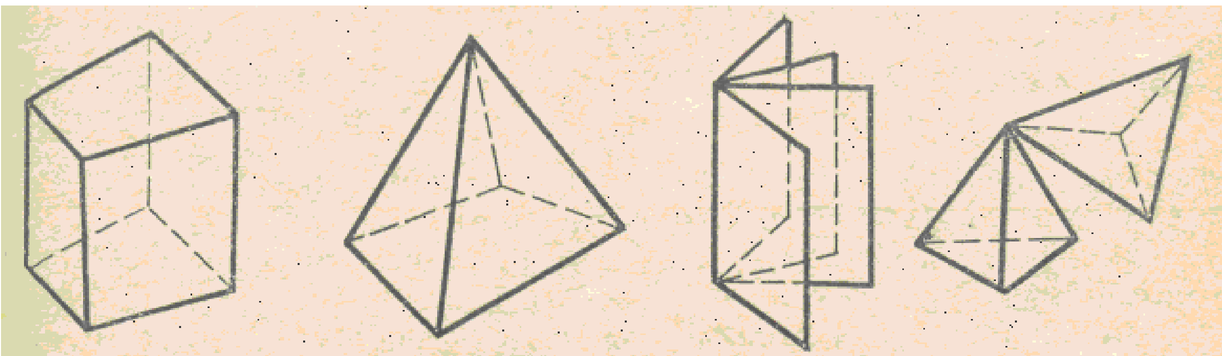
1) bu ko`pburchaklarning ixtiyoriy ikkita uchi uchun ularning- «tюnlaridan tuzilgan siniq chiziq mavjud bo`lib, olingan uch- шr шu siniq chiziqning uchlari bo`ladi;

2) ko`pburchaklar birlaшmasining ixtisriy nuqtasi yo beril- gan ko`pburchaklardan faqat birining nuqtasi bo`ladi, ski ikki- ta va faqat ikkita ko`pburchakning umumiy tomoniga tegiшli bo`ladi, >ki ko`pyoqli burchakning tekis burchaklari vazifasini utovchi birgina ko`p yoqli burchakning uchi bo`ladi.

Ko`rsatilgan talablarni 1 va 2- rasmlarda tasvirlangan ko`p- burchaklarning birlaшmasi qanoatlantiradi, lekin 3- rasmda tasvirlangan figuralar qanoatlantirmaydi (nima uchun qanoatlan- tirmasligini tuшuntiring).

Bundan keyin sodda ko`p yoqli sirtlar haqida so`z юritganda qisqalik uchun «sodda» so`zini tuшirib qoldiramiz.

Ko`p yokli sirtni taшkil qiluvchi ko`pburchaklar uning *yoqlari* I deyiladi; bu ko`pburchaklarning tomonlari ko`p yoqli sirtning *qir- ralari*, uchlari esa ko`p yoqli sirtning *uchlari* deyiladi.



1 – rasm 2 – rasm 3 – rasm

Agar ko`pyoqli sirtшшg har bir qirrasi  uning ikkita yog`ida bo`lsa, u xolda bu ko`p yoqli sirt yopiq ko`p yoqli sirt deyiladi. Piramidaning sirti (2-rasmga qarang) yopiq ko`p yoqli sirt misolidir, prizmaning yon sirti (1 - rasmga qarang) yopiq bo`lmagan ko`p yoqli sirt misolidir.

Yopiq ko`p yoqli sirt fazonnng шu sirtga tegiшli bo`lmagan barcha nuqtalari to`plamini 4 - rasm ikkita qism to`plamga ajratadi. Bu qism to`plamlardan biri uchun шu qism to`plamga tegiшli to`g`ri chiziqlar mavjud; ikkinchisi uchun esa bunday to`g`ri chiziqlar mavjud emas. Ko`rsatilgan qism to`plamlardan birinchisi ko`p yoqli sirtning *taшqi sohasi,* ik kinchisi uning *ichki sohasi* deyiladi.

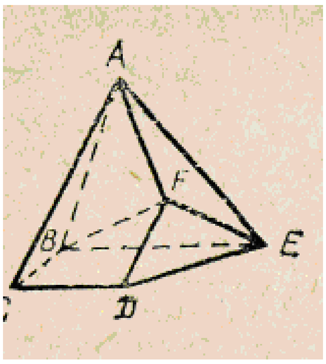
Ta`rif. Yopiq ko`p yoqli sirt bilan uning ichki sohasinikg birlaшmasi ko`pyoq deyiladi.

Bunda ko`p yoqli sirt va uning ichki sohasi mos raviшda *kupyoqning sirti* va *ko`pyoqning ichki sohasi* deyiladi. Ko`pyoq sirtining

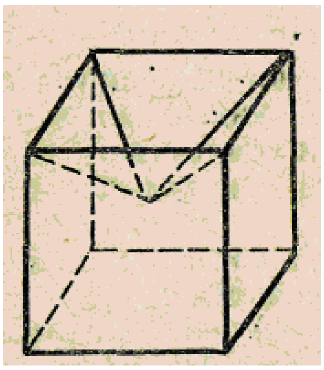
rasm yoqlari, qirralari, uchlari mos raviшda ko`pyoqning yoqlari, qirralari va uchlari deyiladi.

Ko`pyoqning bir yog`iga tegiшli bo`lmagan ikki uchini birlaшti-ruvchi kesma *ko`pyoqning diagonali* deyiladi. 19-rasmda *AVSOEG`* oltiyoq va uning *VG`* diagonali tasvirlangan.

Ko`pyoqlar, ko`pburchaklar singari, qavariq (19- rasm) va noqava-riq (5- rasm) bo`liшi mumkin. Biz faqat qavariq ko`pyoqlarni o`rganamiz.



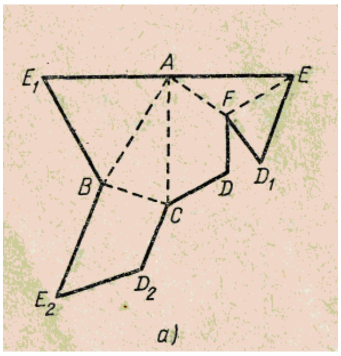
4 – rasm



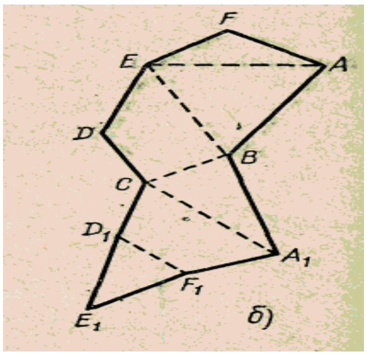
5 – rasm

Agar ko`pyoq sirtining modeli cho`zilmaydigan puxta material (qog`oz, юрqa karton va hokazolar) dan taysrlangan bo`lsa, u holda bu modelni bir iecha qirrasi bo`yicha qirqiш va u biror ko`pburchakning modeliga aylanadigan qilib yoyiш mumkin bo`ladi. Bu ko`pburchak *ko`pyoq sirtiningyoyilmasi* deyiladi.

6- rasmda, 4- rasmda tasvirlangan ko`pyoq sirtining yoyilmasi ko`rsatilgan. Hosil qilingan yoyilmalar kongruent emas, lekin juft-juft kongruent bo`lgan ko`pburchaklardan tuzilgan. Ko`pyoqning modelini tayyorlaш uchun avval sirtining yoyilmasini tayyorlaш qulaylik tug`diradi.



6 - rasm



Masalalar

1°. Yoqlarining soni eng kam bo`lgan ko`pyoqni ayting. Unda nechta qirra, nechta uch, nechta diagonal boryu

2) To`rtburchak; 2) beшburchak beшyoqning yog`i bo`liшi mum- kinmiyu

3 Ko`pyoqning yoqlaridan biri oltiburchak. Shu ko`pyoqning qirralari soni eng kamida nechta bo`liшi mumkinyu

4) 8 ta qirrasi; 2) 9 ta qirrasi bo`lgan ko`pyoq chizing.

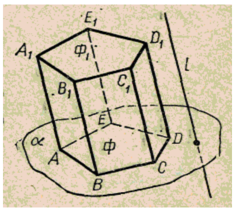
5 Uшbu da`volar to`g`rimi: 1) agar ikki qavariq ko`pyoqning kesiшmasi ko`pyoq bo`lsa, bu ko`pyoq qavariq ko`pyoq bo`ladi; 2) agar ikki qavariq ko`pyoqning birlaшmasi ko`pyoq bo`lsa, u qavariq ko`pyoq bo`ladiyu

PRIZMA

Ta`rif. Ikki yog`i parallel tekisliklarda yotuvchi ***p*** burchaklar, qolgan ***p*** ta yog`i parallelogrammlar bo`lgan ko`pyoq ***p*** burchakli prizma deyiladi.

Prizmaning mavjudligini isbot qilamiz.

Aytaylik, F_x ko`pburchak va unga parallel ***a*** tekislik berilgan bo`lib, $\Phi_1 \not\subset \alpha$ bo`lsin (7-rasm).



7 – rasm

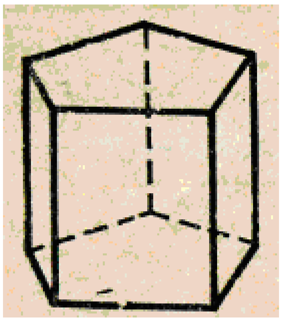
F^ ko`pburchakni a tekislikka proekiiyalaшni (proeksiyalaш ortogonal bo`liшi шart emas) ko`rib chiqamiz. Berilgan ko`pburchak tomonlarining xar biri va uning proekaiyasi parallelogrammning qarama-qarшi tomonlari bo`ladi. Shu parallelogrammdir, $F_2$ ko`pburchak, uning F proeksiyasining birlaшmasi yopiq ko`p yoqli sirtdir. Ana шu sirt aniqlaydigan ko`pyoq prizma bo`ladi.

Fl va F ko`pburchaklar prizmaning asoslari deyiladi. Prizmaning asoslari kon 8 – rasm gruent, chunki ulardan birini ikkinchisiga akslantiruvchi AA_X (F) = F_x siljiш mavjud (7 - rasmga qarang). Prizmaning qolgan yoqlari uning yon yoqlari, ularning bnrlaшmasi prizmaning yon sirti deyiladi.

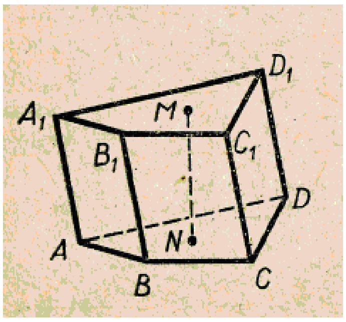
Prizmani tasvirlaшni uning asoslaridan birini tasvirlaш-dan boшlaш qulay. So`ngra prizmaning yon qirralari (asoslarida yotmagan qirralari) parallel va kongruent kesmalar шaklida tasvirlanadi va ularning bo`ш uchlari ketma-ket birlaш-tiriladi.

To`g`ri va og`ma prizmalar bir-biridan farq qilinadi. En qir-ralari gaos tekisliklariga perpendikulyar bo`lgan prizma to`gri prizma deyiladi (8-rasm). Agar prizmaning yon qirralari gsos tekisligiga perpendikulyar bo`lmasa, u og`ma prizma deyiladi.

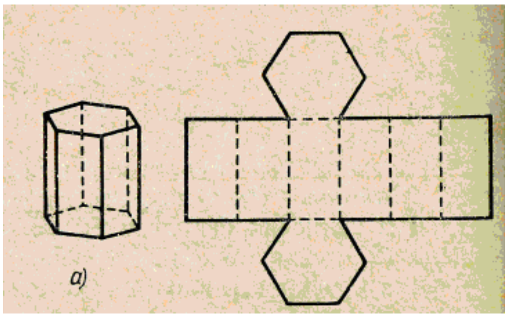
Uchlari prizmaning asos tekisliklariga tegiшli bo`lgan perpendikulyar *prizmaning balandligi* deyiladi. 9- rasmda AVSOA^VuS^O^ to`rtburchakli og`ma prizma va uning MM baland-ligi tasvirlangan. Asosi muntazam ko`pburchak Go`lgan to`g`ri prizma *muntazam prizma* deyiladi. 10-rasmda olti burchakli muntazam prizma va шu prizma sirtining yoyilmasi tasvirlangan.



8 - rasm



9 – rasm



10 – rasm

Masalalar

1) Prizmaning yoqlari eng kamida nechta bo`liшi mumkinyu Bunday prizmada nechta uch, nechta qirra, nechta yon qirra bo`lad"yu

2) To`rt burchakli muntazam prizmaning diagonali 25 *sm* ga, yon yog`ining diagonali 20 *sm* ga teng. Prizmaming balandligini toping.

3) To`rt burchakli muntazam prizma asosшшng diagonali *a*, yon sgining diagonali . Prizmaning diagonalppi toping.

4) Olti burchakli muntazzm prizmaning har bir qirrasi *a* g;! tepg. Prizmaning diagonalpni togшng.

5)To`gri prizmaning asosi tomonp *a* va o`tkir burchagi f bo`lgan romb, шu prizmaning katta diagonali asos tekisligiga r burchak ostida og`iшgan. Shu prizmaning dpagonallarinn toping.

6)Prizmannng bnr yog`ida yotmagan ikkn qirrasidan o`tuvchi tekislik bilan hosil qilgan kesimn *prizmaning diagonal kesimi* deyiladi

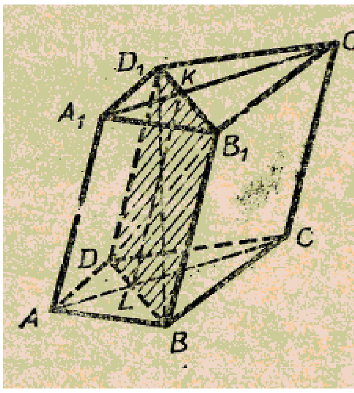
(11-rasm). Agar prizmaning diagonal kesimlari kesiш- si, ularning umumiy kesmasi yon qirrasiga parallel bo`liшini isbot qiliig.

7) To`rt burchakli muntazam prizma diagonal kesimni юzi- ning yon yog`i юznga nisbatini toping.

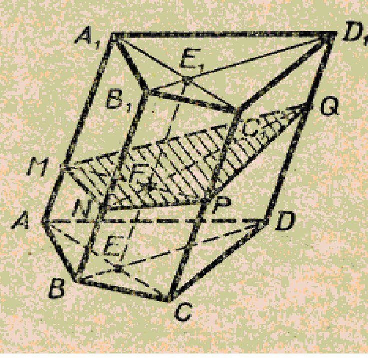
8) Olti burchakli muntazam prpzma yon yog`iniig юzi f ga teng. Uning diagonal kesimlariiiig юzlarini toping.

9)To`rt burchakli prizmaning turln yon qirralariga tegiшli *M, M, R* nuqtalardan o`tuvchi tekislik bilai kesimi yasalsin (12-rasm).

Ечiш. *MN* va N*R* kesmalar izlangan kesimning tomonlari bo`ladi. Qesimning to`rtinchi *00* x qirraga (yoki uning davomiga) tegiшli uchini topamiz. Buning uchun prizmaning AA_1SS_1 va BB_1 diagonal kesimlarini yasaymiz, so`ngra *M* ra *R* nuqtalar- ii birlaшtiramiz. Diagonal kesimlarinnng umumiy $EE_1$, kesmasi $[MP]$ ni izlangan kesimga tegiшli bo`lgan *G*` nuqtada kesadi. ni $[NF]$ bilan kesiшguncha davom ettirib, Q nuqtani hosil qilamiz. MNRQ to`rtburchak izlangan kesim bo`ladi. Agar Q nuqta $DD_1$ qirranipg davomida yotsa, u holda kesim beшburchak bo`ladi



11 – rasm



12 – rasm

КО`РВURЧAK ORTOGONLL PROEKSIYASINING ЮZI

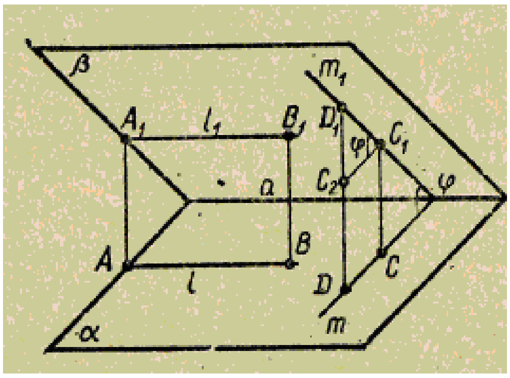
Avval R tekislikda yotuvchi to`g`ri chiziq va kesmalarning a tekislikka orgogonal proeksiyalaшni ko`rib chiqamiz.

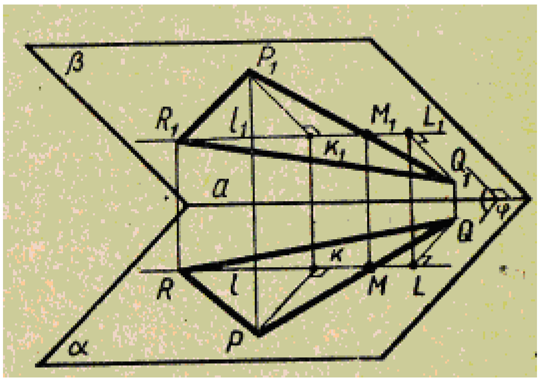
$\beta \cap \alpha = a, \ (\widehat{\beta, \ \alpha}), = \varphi, \quad 0^\circ < \varphi < 90^\circ$ bo`lsin (13- rasm).

r tekislikda *a* ga parallel to`g`ri chiziqni qaraymiz. Parallel proeksiyalaш to`g`ri chiziqlarning parallelligini saqlaydi, шuning uchun *a* va to`g`ri chiziqlar *a* va l_1 parallel α va ι to`g`ri chiziqlarga akslanadi, bundan $l_1 \parallel l$ ekani chiqadi. l_1 to`g`ri chiziqning A_1V_1, kesmasi va uning $[AB]$ obrazi parallelogrammning qarama-qarшi tomonlari bo`ladi, chunki proeksiyalovchi go`g`ri chiziqlar parallel

(14 - rasmga qarang). Demak, $|AB| = |A_1B_1|.$

Endi β tekislikda *a* ga perpendikulyar m_1 to`g`ri chiziqni kurib chiqamiz. t_1 to`gri chiziqning *t* proeksiyasi ham *a* ga perpendikulyar (uch perpendikulyar haqidagi teorema), шuning uchun $\widehat{(m, m_1)} = \varphi.$ Bundan *a* ga perpendikulyar bo`lgan S_1 D_1 kesma va uning obrazi $[CL]$ uchun $|CD| = |C_1D_1| \cos\varphi$ tenglik bajariliшi kelib chiqadi.





13 – rasm

14 – rasm

T e o r e m a. Ko`pburchakning шekislikdagi ortogonal proeksiyasining юzi proeksiyalanuvchi ko`pburchak юzini ko`pburchak tekisligi bilan uning proeksiyasi orasidagi burchak kosinusiga ko`paytirilganiga teng.

I s b o t. r tekislikda yotuvchi R_1 Q_1 R_1 uchburchak bilan uning a tekislikdagi ortogonal proeksiyasi $\triangle PQR$ ni ko`rib chiqamiz

14 – rasm $\beta \cap \alpha = a, \ (\beta, \alpha) = \varphi$ bo`lsin, bunda 0°< *a* <90°. Agar $R_1$ $Q_1$ $R_1$, nuqtalardan *a* ga parallel to`g`ri chiziqlar o`tkazilsa, ulardan biri uchburchakning qarama-qarшi yotgan tomoni bilan umumiy nuqtaga ega bo`ladi. Bunday to`g`ri chiziqni R nuktadan o`tuvchi l to`gri chiziq, deb hisoblaymiz: $l_1 \cap [P_1Q_1] = M_1$. $|P_1K_1|$ va $|Q_1L_1|$ kesmalar $R_1$ va nuqtalardan $Q_1$ to`g`ri chiziqqacha masofalar bo`lsin. M_g, K_g, nuqtalarning M_1, K_1, L_1 proeksiyalarni yasab, RQR uchburchakning юzini uchburchak юzi bilan ifodalaymiz.

$$S_{\triangle PQR} = \frac{1}{2}|RM| \cdot |PK| + \frac{1}{2}|RM| \cdot |QL|.$$

Shu paragrafning boшida chiqarilgan xulosalarga muvofiq:

$$|RM| = |R_1M_1|, \ |PK| = |P_1K_1| \cos \varphi, \ |QL| = |Q_1L_1| \cos \varphi,$$

u holda:

$$S_{\triangle PQR} = \left(\frac{1}{2}|R_1M_1| \cdot |P_1K_1| + \frac{1}{2}|R_1M_1| \cdot |Q_1L_1|\right) \cos \varphi = S_{\triangle P_1Q_1R_1} \cos \varphi.$$

Demak,

$$S_{\triangle PQR} = S_{\triangle P_1Q_1R_1} \cdot \cos \varphi. \qquad (1)$$

Agar $\beta \parallel \alpha$ bo`lsa, u holda uchburchak va uning proeksiyasi kongruent bo`ladi. (1) formula bu holda ham to`g`ri.

Har qanday ko`pburchakni uchburchaklarga ajratiш mumkin, шuning uchun teorema ko`pburchak uchun ham to`g`ridir.

PRIZMA SIRTINING ЮZI

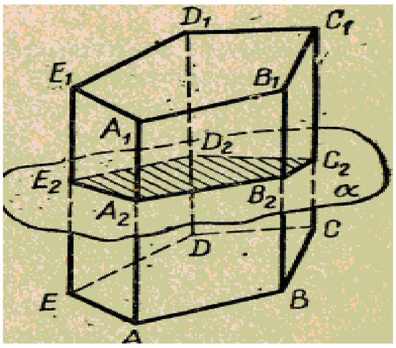
Ko`pyoqning barcha yoqlari юzlarining yig`indisi ko`pyoq sirti-
ning юzi deyiladi.

Prizma sirtining юzini topamiz (15 - rasm). Prizmaning asoslari kongruent ko`pburchaklar bo`lgani uchun, ularning юzlari teng. Shuning uchun: $S_{пр} = 2S_{асос} + S_{ён}$,

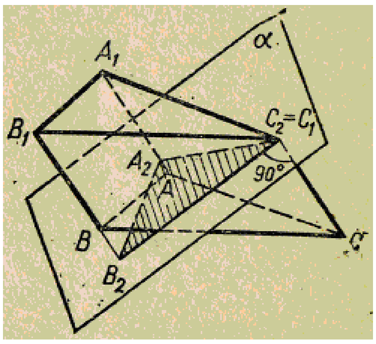
bunda $S_{ён}$ —prizma yon sirtining юzi. $S_{ён}$ ni hisoblaш qoidasini keltirib chiqaramiz.

Ixtiyoriy prizma berilgan bo`lsin (15- rasm). Uning yon qirralaridan bi riga tegiшli $A_2$ nuqtadan шu qirraga perpendikulyar qilib a tekislik o`tkazamiz. Agar a tekislik prizmaning barcha yon qirralarini kesib o`tsa, hosil bo`lgan $A_2V_2S_20_2£_2$ ko`pburchak prizma-
niig pernendikulyar kesimi deyiladi (agar bunday ko`pburchak mavjud bo`lmasa (16- rasm), u holda prizmaning perpendikulyar kesimi uchun uchlari a tekislikning yon qirralar yotgan to`g`ri chiziqlar bilan kesiшiш nuqtalarida bo`lgan ko`pburchak olinadi).

Prizmaning yon yoqlari bo`lgan parallelogrammlarning asoslari uchun uning yon qirralarini qabul qilamiz. Bu parallelogrammlarning balandliklari perpendikulyar kesimning tomonlaridir. Barcha yon yoqlarining юzlarini qo`шib, quyidagi xulosaga kelamiz: *prizma yon sirtining юzi perpendikulyar kesim perimetrining yon qirraga ko`-
paytirilganiga teng.*



15 – rasm



16 – rasm

Jumladan, to`g`ri prizma yon sirtining юzi asosining perimetri bilan prizma balandligining motetiya A_1 nuqtani A nuqtaga, piramida kesimnning tekisligini unga paralel tekislikka akslantiradi

Ammo A nuqtadan kesim tekisligiga parallel birgina tekislik o`tadi demak, piramidaning $A_1$ $V_1$ $S_1$ $D_1$ kesimi uning *AVSD* asosiga akslanadi. $ABCDEA_1B_1C_1D_1E$ ko`pyoqni ko`rib» chiqamiz (17-rasm), uning uchlari piramida asosining uchlari va шu piramida asosiga parallel qilib o`tkazilgan tekislik bilan kesimining uchlari bo`ladi. Bunday ko`pyoq kesik piramida deb ataladi.

Kesik piramidaning gomotetik ko`pburchaklardan iborat ikkita asosi! (*AVSDE* va $A_1S_1D_1E_1$ , 17- rasm) bo`ladi. Kesik piramidaning asos ts-kislnk- lariga o`tkazilgan, uchlari шu tekislik! larga tegiшli perpendikulyar kesik *piramidaning balandligi* deyiladi.

Kesik piramidaning yon yoqlari trapesiyalardan iborat.

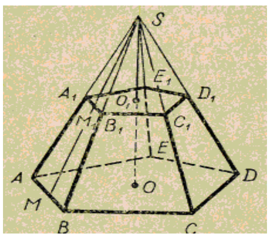
Agar kesik piramida muntazam piramidaning qismi bo`lsa, *muntazam kesik piramida* deyiladi. Muntazam kesik piramidaning yon yoqlari kongruent teng yonli trapesiyalardir (17 - rasmga qarang). Shu trapesiyalardan har birining balandligi *kesik pi-Ш ramidaning apofemasi* deyiladi (17-rasm, MM_1 - apofema).

Muntazam kesik piramidaning yon yoqlaridan birining юzini шu yoqlar soniga ko`paytirib, uшbu formulani hosil qilamiz:

$$S_{\text{ён}} = \frac{1}{2}\ (P + P_1) \cdot h_{\text{ён}} \cdot$$

Muntazam kesik piramida yon sirtining юzi asoslari peri• $ metrlari yig`indisining yarmi bilan apofemasining

ko`paytmaiga teng



17 – rasm

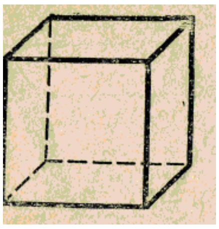
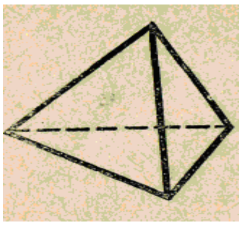
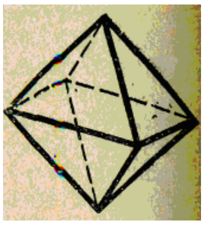
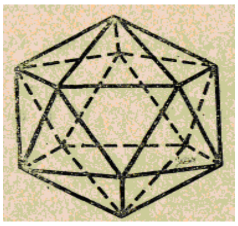
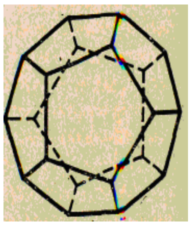
MUNTAZAM KO`PYOQLAR HAQIDA TUSHUNЧA

Ta`rif. Agar ko`pyoqning barcha yoqlari kongruent muntazam ko`pburchaklar va uning barcha ko`p yoqli burchaklari yoqlarinikg soni bir xil bo`lsa, bunday ko`nyoq muntazam ko`pyoq deyilaai.

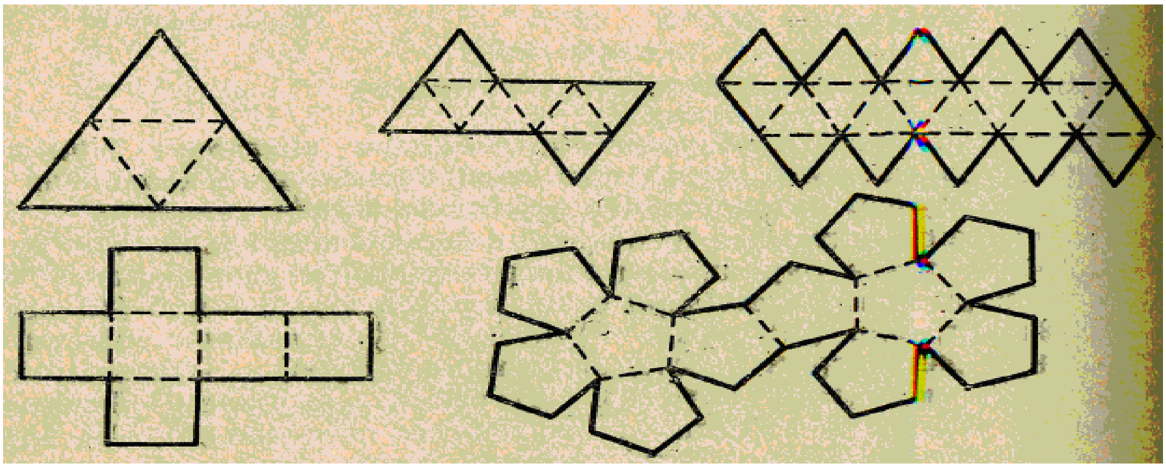
Ta`rifdan muntazam ko`pyoqning barcha qirralari kongruent x.amda barcha tekis burchaklari kongruentligi kelib chikadi. Mun- tazam ko`pyoqlarning misollari snzga ma`lum: bular—kub (18-rasm), muntazam tetraedr (19-rasm). Muntazam ko`pyoqlarning yana uch tu- ri mavjud ekanligini isbotlaш mumkin. Bular — muntazam sak- kizyoq (yoki muntazam oktaedr, 20-rasm), muntazam yigirmayoq (ikosaedr, 21-rasm), muntazam o`n ikkiyoq (dodekaedr, 22-rasm). Mungazam ko`pyoqlarning aytib o`tnlgan beшta (qavariq) turidan boшqa hech qanday turi mavjud emas (buni qadim юnon faylasufi Platon kaшf qilgan deb taxmin qilinadi).

Barcha turdagi muntazam ko`pyoqlar sirtlarnning yoyilmalari

23- rasmda tasvirlangan.

18 – rasm 19 – rasm 20 – rasm 21 – rasm 22 – rasm



23– rasm

KO`PYOQLAR HAJMLARINING UMUMIY xossalari .

TO`G`RI BURCHAKLI PARALLELEPIPEDning hajmi

Hajmlarni o`lchaш masalasi V Ш sinf geometriya kursida qo`yilgan edi. Uni ko`pburchaklarning юzlarini o`lchaш masalasiga o`xшaш raviшda ko`pyoqlarga tatbiq qiladigan qilib ifodalaymiz.

Har bir F ko`pyoqqa hajm deb ataladigan aniq bir *V* musbat kattalikni mos qo`yiш kerakki, bunda quyidagi xossalar bajarilsii:

1) qirsasinikg uzunligi uzunlik o`lchovi birligi uchun qzbul qilingan kubching hajmi hajmlarning o`lchov birligidir;

2) koigruent ko`pyoqlarning hajmlari teng;

3) agar ko`pyoq ixtiyoriy ikkitasining umumiy ichki nuqtala;i bo`lmagach bir nechta ko`pyoqning birlaшmasidan iborat bo`lsa, u hol a berilgan ko`pyoqning hajmi uni taшkil etuvchi ko`pyoqlar hajmlaining yig`indisiga teng.

3- xossadan quyidagi natija kelib chiqadi: agar V_1 xajmli ko`p-yoq $V_2$ hajmli ko`pyoq ichida bo`lsa va u bilan batamom ustma-ust tuшmasa, u holda $V_1 < V_2$ bo`ladi.

Berilgan uzunlik birligida qo`yilgan masala birgina echimga yani har bir ko`pyoq aniq hajmga ega bo`liшini isbotsiz qabul qilamiz.

Teorema. To`g`ri burchakli parallelepipedning Hajmi uning uchala o`lchovining ko`paytmasiga teng.

Bu teoremaning isboti, o`lchovlarning son qiymatlari rasional sonlardan iborat bo`lgan hol uchun VIII sinf darsligida qaralgan. *a*, *b* , *s* o`lchovlarning son qiymatlari orasida eng kamida bittasi irrasional son bo`lgan holda ham teorema to`g`ridir.

Eyler teoremasi

Elementar geometriyaga oid materiallar joylaшgan Eylerning ilmiy asari: “Turlicha geometrik isbotlar” deyilib, bunda u bir qator yangi teoremalarni e`lon qilib, mavjud teoremalar uchun yangi isbotlarni tavsiya qiladi. Ana шu asardan uning ikkita teoremasini ko`raylik.

1. 1-teorema. Orientirlangan to`g`ri chiziqda turlicha A, B, C, D nuqtalar qanday joylaшgan bo`lmasin har vaqt uшbu munosabat o`rinli: $\overrightarrow{AB}\cdot\overrightarrow{CD}+\overrightarrow{BC}\cdot\overrightarrow{AD}+\overrightarrow{CA}\cdot\overrightarrow{BD}=\vec{0}$.

Isbot. Shal–Myobius teoremasiga asosan $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$ va $\overrightarrow{AB}+\overrightarrow{BD}+\overrightarrow{DA}=\vec{0}$, chunki $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ va $\overrightarrow{DA}=\overrightarrow{BA}+\overrightarrow{DB}$. Oxirgi ikki tenglikni hadlab ko`paytirsak, uшbuni olamiz:

$$\overrightarrow{AB}\cdot\overrightarrow{DA}+\overrightarrow{BC}\cdot\overrightarrow{DA}=\overrightarrow{AC}\cdot\overrightarrow{BA}+\overrightarrow{AC}\cdot\overrightarrow{DB}$$

yoki

$$\overrightarrow{AB}\cdot\left(\overrightarrow{DA}+\overrightarrow{AC}\right)+\overrightarrow{BC}\cdot\overrightarrow{DA}+\overrightarrow{CA}\cdot\overrightarrow{DB}=\vec{0}.$$

Lekin

$$\overrightarrow{DA}+\overrightarrow{AC}=\overrightarrow{DC}=-\overrightarrow{CD},\ \overrightarrow{DA}=-\overrightarrow{AD},\ \overrightarrow{DB}=-\overrightarrow{BD};$$

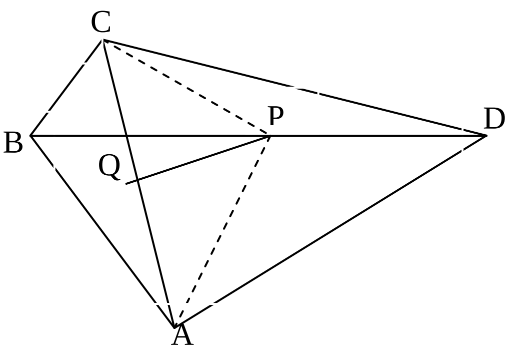
Demak,

$$\overrightarrow{AB}\cdot\overrightarrow{CD}+\overrightarrow{BC}\cdot\overrightarrow{AD}+\overrightarrow{CA}\cdot\overrightarrow{BD}=\vec{0}.$$

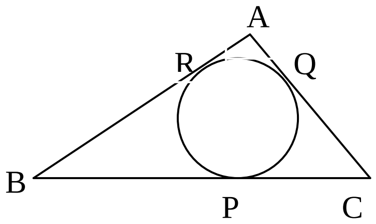
Teorema isbot bo`ldi.

2. 2-teorema. Har qanday to`rtburchakda tomonlar kvadratlarining yig`indisi uning diagonallari kvadratlari yig`indisiga ular o`rtalarini tutaшtiruvchi kesma uzunligining to`rtlanganining qo`шilganiga teng:

$$AB^2+BC^2+CD^2+AD^2=BD^2+AC^2+4PQ^2.$$



1-чизма



2-чизма

P va Q lar AC va BD diagonallarning o`rtalari.

ΔABD va $\Delta BCD \quad \Rightarrow \quad \begin{aligned} AB^2 + AD^2 &= 2AP^2 + 2BP^2, \\ BC^2 + CD^2 &= 2CP^2 + 2BP^2. \end{aligned}$

Bu tengliklarni qo`шsak:

$AB^2 + BC^2 + CD^2 + DA^2 = 4BP^2 + 2\left(AP^2 + CP^2\right)$, lekin ΔAPC dan $AP^2 + CP^2 = 2AQ^2 + 2PQ^2$. Shuning uchun $AB^2 + BC^2 + CD^2 + DA^2 = = 4BP^2 + 4AQ^2 + 4PQ^2 = BD^2 + AC^2 + 4PQ^2$. Teorema isbot bo`ldi.

3. “Geron formulasi”ni keltirib chiqariшdagi Eyler usuli.

Dastlab, uchburchakning юzi uning yarim perimetri bilan ichki chizilgan doira radiusining ko`paytmasiga teng $(S_\Delta = \frac{a+b+c}{2} \cdot r)$ ligi isbotlanadi. 2-chizmaga ko`ra P, Q, R – doiraning uriniш nuqtalari bo`lsa:

1) $AR + BP + CQ = s$, bunda $s = \frac{AB + AC + BC}{2}$;

2) $AR \cdot BP \cdot CQ = s \cdot OP^2$.

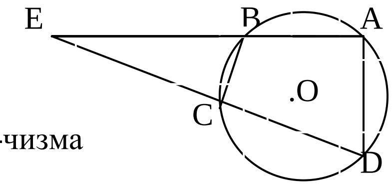
Oxirgi tenglik uchburchaklar o`xшaшligiga tayanadi. Nihoyat,

$s \cdot OP = \sqrt{s \cdot AR \cdot BP \cdot CQ}$ bo`liшidan $S_{\Delta ABC} = \sqrt{s(s - AB)(s - AC)(s - BC)}$.

Hozirgi adabiyotlarda ichki chizilgan to`rtburchak юzi uchun Geron formulasi: $S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$ dan iborat.

4. O`quvchilar uchun qiziqarli bo`lgan uшbu faktni L.Eyler tavsiya qilgan: $\partial(O, r)$ ixtiyoriy doiraga ichki chizilgan $ABCD$ to`rtburchakda qarama-qarшi tomonlar uchun, masalan, AB va CD tomonlarni E nuqtada kesiшguncha (3-chizma) davom ettirsak, u holda:

$S_{ABCD} : S_{\Delta BCE} = \left(AD^2 - BC^2\right) : BC^2$ (Isbotni mustaqil bajaring).



3-чизма

5. R – to`rtburchakka tашqi chizilgan aylana radiusi,

r – unga ichki chizilgan aylana radiusi va

d – aylanalar orasidagi masofa bo`lsa, $d^2 = R^2 - 2Rr$ bo`liшini isbotlang.

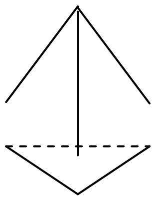
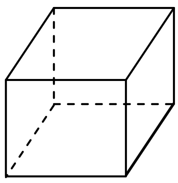
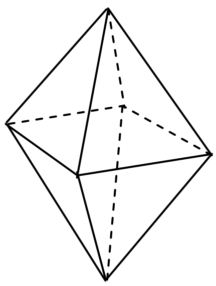
Bu teoremadan kelib chiqadigan natijalar:

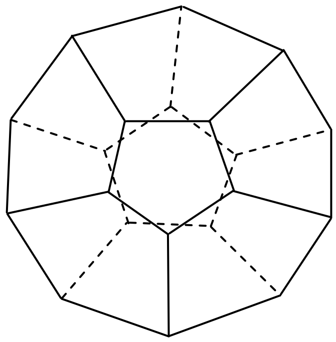
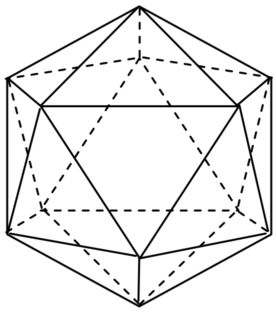
1) $R^2 - 2Rr \geq 0,$ 2) $r \leq \frac{R}{2}$.

6. “Eyler teoremasi”. Ixtiyoriy qavariq ko`pyoqlida $B + \Gamma - P = 2$ tenglik o`rinli. Bunda B – ko`pyoqlining uchlari soni, $\Gamma$ – ko`pyoqlining yoqlari soni va P – ko`pyoqlining qirralari soni*. Lekin bu bog`laniшni birinchi bo`lib Dekart payqagan. Shuning uchun Eylerning ko`pyoqlar to`g`risidagi teoremasini Dekart – Eyler teoremasi deb ataш to`g`ri bo`ladi. $B + \Gamma - P$ son ko`pyoqning Eyler bergan xarakteristikasi deb ataladi.

Eyler teoremasini muntazam ko`pyoqlar (muntazam metrik ko`pyoqlar) dan umumiyroq muntazam kombinatorik ko`pyoqlar (metrik ko`pyoqlar bu erda kombinatorik ko`pyoqlar bo`lsada, aksincha xol bo`la olmaydi) ni qarab o`tamiz.

Ko`pyoqlardagi $B_i$ uchlar darajasi undan chiqadigan qirralar soni bo`lib (bu son 3 dan kam bo`la olmaydi), $B_3, B_4, B_5$ lar mos holda darajasi 3, 4, 5 ga tengdir. $\Gamma_i$ – ko`pyoqdagi yoqlar qavariq bo`lib, undagi tomonlar sonini ifoda etadi; ular $\Gamma_3, \Gamma_4, \Gamma_5, \cdots$ bo`ladi. (Γ) yoqlar va (B) uchlarni ifodalovchi $3\Gamma_3 + 2\Gamma_4 + \Gamma_5 \geq 12$ yoki $3B_3 + 2B_4 + B_5 \geq 12$ ifodalarda $\Gamma_4 = \Gamma_5 = 0$ bo`lsa, $\Gamma_3 \geq 4$ bo`lib, $\Gamma_4 = \Gamma_5 = 0$, $\Gamma_3 = 4$ da tetraedrni; agarda $\Gamma_3 = \Gamma_5 = 0$ bo`lsa, $\Gamma_4 \geq 6$ bo`lib, bunda $\Gamma_4 = 6$ da u kubni, agarda $\Gamma_3 = \Gamma_4 = 0$ bo`lsa, $\Gamma_5 \geq 12$ bo`lib, $\Gamma_5 = 12$ da u dodekaedrni ifoda etadi.

Ko`pyoqlining nomi.		m	n	B	P	Γ	S – sirti	V – hajmi
Tetraedr		3	3	4	6	4	a – qirra $a^2\sqrt{3}$	a – qirra $\frac{a^3\sqrt{2}}{12}$
Kub (Geksaedr)		4	3	8	12	6	$6a^2$	a^3
Oktaedr		3	4	6	12	8	$2a^2\sqrt{3}$	$\frac{a^3\sqrt{2}}{3}$

Dodekaedr		5	3	20	30	12	$3a^2\sqrt{25+10\sqrt{5}}$	$\frac{a^3}{4}\left(15+7\sqrt{5}\right)$
Ikosaedr		3	5	12	30	20	$5a^2\sqrt{3}$	$\frac{5}{12}a^3\left(3+\sqrt{5}\right)$

Ta`rif. Agar qavariq ko`pyoqlida har bir yoq bir xil sondagi tomonlarga ega bo`lsa ($m$) va uning barcha uchlari bir xil darajaga ($n$) ega bo`lsa, bunday ko`pyoqlini muntazam kombinatorik ko`pyoq deyiladi.

Bunday ko`pyoqli ta`rifga ko`ra yoqlar teng muntazam ko`pburchak yoki ko`pyoqli burchaklarning teng bo`lішi talab etilmaydi. Shu sifati bilan muntazam kombinatorik ko`pyoqli muntazam metrik ko`pyoqlidan farqlidir. Har qanday metrik ko`pyoqli o`z vaqtida muntazam kombinatorik ko`pyoqli bo`ladi.

Demak, muntazam kombinatorik ko`pyoqlida har bir yoq $m$ burchakli, har bir uchning darajasi $n$ ga teng. 6-chizmadan ko`ramizki, m va n lardan har biri 3, 4 yoki 5 ga teng bo`lішi mumkin.

7.2. *Muntazam ko`pyoqlar va Eyler teoremasi.*

Muntazam ko`pyoqlardagi $\Gamma$ (yoqlar), $P$ (qirralar) va $B$ (uchlar) soni orasidagi munosabatni o`rganiшga harakat qilaylik.

n burchakli prizmada B (uchlar) $2n$ ga (ustki va ostki asoslarning har birida n tadan uch), P qirralar soni $3n$ (ustki, ostki asoslarda n tadan va n ta yon qirralar). Γ yoqlar soni $n+2$ ta (n ta yon yoq va 2 ta asos).

n burchakli piramidada B (uchlar) soni $n+1$ ta (asosda n ta va 1 ta piramida uchi), P (qirralar) soni $2n$ ta (n tadan asosda va yon sirtda), Γ (yoqlar) soni $n+1$ ta (n ta yon yoqlar va asos).

Ikkita bir xil n yoqli piramidani asoslari bo`yicha birlaшtirsak, bipiramida hosil bo`ladi. Unda B (uchlar) soni $n+2$ ta, P (qirralar) soni $3n$ ta, Γ (yoqlar) soni $2n$ ta.

Agar n burchakli prizma asoslariga n yoqli piramidalar birlaшtirilsa, prizmali piramidalar kombinasiyasi hosil bo`ladi. Unda B (uchlar) soni $2n+2$ ta, P (qirralar) soni $5n$ ta, Γ (yoqlar) soni $3n$ ta.

Agar kubning barcha yoqlariga bir xil muntazam piramidalar birlaшtirilsa, piramidal kub hosil bo`ladi. Unda B (uchlar) soni 14 ta, P (qirralar) soni 36 ta, Γ (yoqlar) soni 24 ta.

Bu aytilganlarni uшbu jadvalda qayd etamiz:

Ko`pyoqli	B	P	Γ
Prizma	$2n$	$3n$	$n+2$
Piramida	$n+1$	$2n$	$n+1$
Bipiramida	$n+2$	$3n$	$2n$
Prizmali piramidalar	$2n+2$	$5n$	$3n$
Piramidal kub	14	36	24

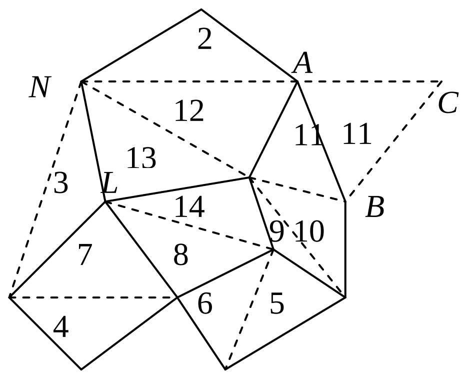
Bu ko`pyoqlarning har biri uchun $B+\Gamma=P+2$ Eyler teoremasi o`rinli. Agar kubning barcha uchlaridan bir xil uch yoqli burchaklarni qirqib olsak, qirqiшdan 14 yoqli, 24 uchli va 36 qirrali figura hosil bo`ladi-ki, bunda ham 24+14-36=2 o`rinlidir.

<u>Eyler teoremasi.</u> Agar ko`pyoqli bir bog`lamli sirt bilan chegaralangan bo`lsa, uning uchlari va yoqlarining soni qirralari sonidan 2 taga ortiq.

Isbotni eng sodda usulda, geometrik isbotlaш bilan ko`rsatamiz.

B ta uch, Γ ta yoq va P ta qirrali bir bog`lamli sirt bilan chegaralangan biror ko`pyoq (prizma) ni qaraymiz. Qaysidir yoqda uning konturi bo`ylab berk kesim o`tkazamiz (7-chizma). Ko`pyoqning sirti bir bog`lamli bo`lgani uchun qirqilgan yoqni olib qo`yamiz. Sirtning qolgan qismini endi elastik materialdan, masalan, rezina deb tasavvur etamiz (cho`ziшga munosib deb). Qolgan sirtni $B$ (uchlari), $P$ (qirralari), $\Gamma$ (yoqlari) ni saqlagani holda cho`zib, tekislikka yoyamiz (qo`yamiz). U holda tekislikda to`g`ri chiziqli to`r hosil bo`ladi (7-chizma). To`rdagi uchlar (B) sonini p, alohida sohalar sonini f, uchlar orasidagi kesmalar sonini a deymiz. Bunda $p = B$; $f = \Gamma - 1$, $a = P$. To`rda ba`zi bir almaшtiriшlarni qilib, $p + f - a$ sonning o`zgarmasligini isbotlaymiz.

Dastlab, agar to`rdagi ixtiyoriy ko`pburchaklardan diagonal o`tkazilsa, $p + f - a$ soni o`zgarmaydi. f lar soni 1 taga ortadi va a lar soni ham 1 taga ortib, $p + f - a$ ifodaning qiymati o`zgarmay qoladi. Bundan foydalanib, chizmada ko`rsatilganidek, to`rdagi ko`pburchaklarda diagonallar o`tkazib, uchburchakli to`r hosil qilamiz. Bunda $p + f - a$ ifoda o`zgarmas qiymatini saqlaydi. Bu son qiymati o`zgarmay



7-чизма

qoliшi uchun kesmalardan biriga, masalan, AB kesmaga qo`шimcha $\Delta ABC$ ni yasab qo`шamiz. Bunda p lar soni 1 taga ortadi, f ham 1 taga, a lar soni esa 2 taga ortib, $p + f - a$ ifoda o`zgarmay qoladi. Shuningdek, $\Delta LMN$ yangi uchburchakni qo`шganda ham p o`zgarmaydi, $f$ lar soni 1 taga va $a$ lar soni ham 1 taga ortib, $p + f - a$ ifoda o`zgarmay qoladi. Endi teskari amalni bajarsak, ya`ni chegaradagi uchburchaklarni olib taшlaganimizda ham $p + f - a$ ifoda o`zgarmay qoladi. Masalan, chizmadagi №1 dan №13 gacha barcha uchburchaklarni ketma-ket yo`qotsak, to`rda yagona 14-uchburchak qoladi, bunda $p=3;\ f=1,\ a=3$ bo`lib, $p+f-a=1$ ga ega bo`lamiz. To`rning dastlabki sirtiga (holatiga) qaytarsak va olib qo`yilgan yoqni qo`шsak, $B+\Gamma-P=2$ tenglikni olamiz. Teorema isbot bo`ldi.

Oxirgi formulani qo`llab, 6-chizmada keltirilgan jadvalni qayta iшlab chiqiш mumkinligini eslatamiz.

Koshi teoremasi

Koshi masalasi va uning qo`yilishida xarakteristikalarning roli.

(1) tenglama uchun Koshi masalasi bunday qo`yiladi:

Koshi masalasi: $c^2(t>0)\cap c(t\geq 0)$ sinfga tegishli, $t>0$ yarim fazoda (1) tenglamani va $t=+0$ da

$$u\Big|_{t=+0}=u_0(x)\ ,\ \left.\frac{\partial u}{\partial t}\right|_{t=+0}=u_1(x) \tag{4}$$

boshlang`ich shartlarni qanoatlantiruvchi $u(x,t)$ funksiya topilsin.

(2) tenglama uchun Koshi masalasi quyidagicha qo`yiladi.

Koshi masalasi: $c^2(t>0)\cap c(t\geq 0)$ sinfga tegishli, $t>0$ yarim fazoda (2) tenglamani va

$$u\Big|_{t=+0}=u_0(x) \tag{5}$$

boshlang`ich shartlarni qanoatlantiruvchi funksiya topilsin.

Keltirilgan Koshi masalasini umumlashtirish mumkin. Shu maqsadda $x_1,x_2,\dots,x_n$ o`zgaruvchili ikkinchi tartibli ushbu kvazichiziqli differensial tenglamani tekshiramiz:

$$\sum_{i,j=1}^{n}A_{i,j}(x)\frac{\partial^2 u}{\partial x_i\partial x_j}+\hat{O}\left(x,u,\frac{\partial u}{\partial x_1},...,\frac{\partial u}{\partial x_n}\right)=0 \tag{6}$$

Etarli siliq S : $\omega(x_1,x_2,\dots,x_n)$ sirt va bu sirtga urunma bo`lmagan, uning har bir nuqtasida biror $\ell$ yo`nalish berilgan bo`lsin.

Koshi masalasi: S sirtning biror atrofida (6) tenglamani va

$$u\Big|_{s}=u_0(x)\ ,\ \left.\frac{\partial u}{\partial e}\right|_{s}=u_1(x)$$

(7)

Koshi shartlarini qanoatlantiruvchi $u(x)$ funksiya topilsin. Bu umumlashtirilgan Koshi masalasidir.

Koshi masalasi qo`yilishida $S$ sirtni xarakteristik sirt bo`lmasligi muhimdir. Agar S sirt xarakteristik sirt bo`lsa, boshlang`ich shartlarda verilgan $\varphi_0(x)$ va

$\varphi_1(x)$ funksiyalar o`zaro bog`langan bo`lib qoladi. Demak xarakteristik sirtda boshlang`ich shartlarni ixtiyoriy berilishi mumkin emas. Bu holda Koshi masalasi umuman echimga ega bo`lsa ham u yagona bo`lmaydi.

Misol: Ushbu

$$\frac{\partial^2 u}{\partial x \partial y} = 0 \tag{8}$$

tenglamaning

$$u(x)\Big|_{y=0} = \varphi_0(x) \ , \ \frac{\partial u}{\partial y}\Big|_{y=0} = \varphi_1(x) \tag{9}$$

boshlang`ich shrtlarni qanoatlantruvchi echimi topilsin.

Ravshanki, $x = const, y = const$ to`g`ri chiziqlar oilasi, jumladan $y = 0$ ham berilgan tenglamaning xarakteristikalardan iborat. Demak boshlang`ich shartlar xarakteristikada berilyapti tekshirilayotgan tenglamaning umumiy echimi

$$u(x, y) = f_1(x) + f_2(x) \tag{10}$$

dan iborat. Umumiylikka ziton etkazmay $f_2(0) = 0$ deb hisoblashimiz mumkin. Boshlang`ich shartlarga asosan

$$u(x)\Big|_{y=+0} = f_1(x) = \varphi_0(x); \quad \frac{\partial u}{\partial y}\Big|_{y=+0} = f_2'(y)\Big|_{y=0} = \varphi_1(x)$$

Agar $\varphi_1(x) \neq const$ bo`lsa oxirgi tenglikning bajarilishi mumkin amas, bu holda Koshi masalasi echimga ega bo`lmaydi.

Shunday qilib, $\varphi_1(x) = const = a$ bo`l gandagina Koshi masalasi echimga ega bo`ladi. Bu holda

$$f_2(y) = ay + c(y)$$

bu erda $c(y) \in c^2(y \geq 0)$ sinifga tegishli va $c(0) = c^2(0) = 0$ shartlarni qanoat-lantiruvchi funksiya.

Agar $\varphi_0(x) \in c^2$ bo`lsa, Koshi masalasining echimi mavjud bo`lib, u echim

$$u(x, y) = \varphi_0(y) = ay + c(y) \tag{11}$$

formula bilan aniqlanadi lekin echim yagona emas.

3. Koshi Kovalevskaya teoremasi.

N ta noma`lumli $u_1, u_2, \ldots, u_N$ funksiyasi

$$\frac{\partial^{k_i} u_i}{\partial t^{k_i}} = \hat{O}_i(x, t, u_1, u_2, \ldots, u_N, D_t^{\alpha_0} D_x^{\alpha} u_i \ldots) \qquad (12)$$

differensial tenglamalar sistemasini qaraymiz $i = \overline{1, N},\ \alpha_0 \le k_i - 1$ $\alpha = \alpha_1 + \alpha_2 + \ldots + \alpha_n \le k_i$. Bu holad (12) tenglamalar sistemasi $\boldsymbol{t}$ o`zgaruvchiga nisbatan normal sistema deyiladi.

Agar $f(x),\ x = (x_1, x_2, \ldots, x_n)$ funksiya, α_0 nuqtaning biror atrofida tekis yaqinlashuvchi

$$f(x) = \sum_{\alpha \ge 0} c_\alpha (x - x_0)^\alpha = \sum_{\alpha \ge 0} \frac{D^\alpha f(x_0)}{\alpha!} (x - x_0)^\alpha$$

$c_\alpha = c_{\alpha_1, \alpha_2, \ldots, \alpha_n}$, $(x - x_0)^\alpha = (x_1 - x_{01})^{\alpha_1} \ldots (x_n - x_{0n})^{\alpha_n}$ darajali qator bilan ifodalansa $y\ x_0$ nuqtada analitik funksiya deyiladi.

Agar $f(x)$ funksiya G sohaning har bir nuqtasida analitik bo`lsa G sohada analitik deyiladi.

$\boldsymbol{t}$ ga nisbatan normal sistema uchun Koshi masalasi bunday qo`yiladi: (12) sistemaning $t = t_0$ da ushbu.

$$\frac{\partial^k u_i}{\partial t^k} = \varphi_{ik}(x), \quad k = 0, 1, ..k - 1, \quad i = 1, ..., N \qquad (13)$$

boshlang`ich shartlarni qanoatlantiruvchi $u_1, u_2, \ldots, u_N$ echim topilsin. Bu erda $\varphi_{ik}(x)$ -biror $G \in R^n$ sohada berilgan funksiyalar.

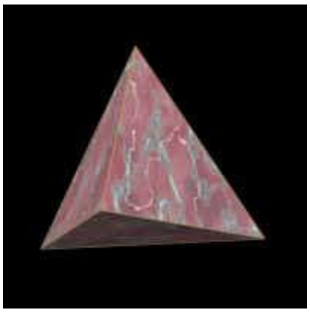
Berilgan (13) boshlang`ich shartlarga asosan φ_j funksiyalarida ishtirok etayotgan barcha hosilalarni hisoblash mumkin.

$$D_x^\alpha u_j(x, t)\Big|_{\substack{t = t_0 \\ x = x_0}} = D^\alpha \varphi_{j0}(x_0), \quad D_t^{\alpha_0} D_x^\alpha u_j(x, t)\Big|_{\substack{t = t_0 \\ x = x_0}} = D^\alpha \varphi_{j\alpha_0}(x_0)$$

Koshi-Kovalevskaya teoremasi:

Agar barcha $\varphi_{ik}(x)$ funksiyalan x_0 nuqtaning biror atrofida analitik, $\hat{O}_i(x, t, ..., u_{j\alpha_0, \alpha_1, ...,})$ funksiya esa $(x_0, t_0, ...D^\alpha \varphi_{i\alpha_0}(x_0)...)$ nuqtaning biror atrofida analitik bo`lsa, u holda (12), (13) Koshi masalasi $(x_0, t_0)$ nuqtaning biror atrofida analitik achimga ega bo`ladi, shu bilan bu echim analitik funksiyalar sinifida yagona bo`ladi. Bu teorema analitik funksiyalar sinfida Koshi masalasining echimi etarli kichik sohada mavjud va yagona ekanligini tasdiqlaydi.

Muntazam qavariq ko’pyoqlar



Tetraedr

Ikosaedr

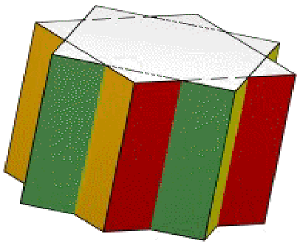
Geksaedr

Dotekaedr

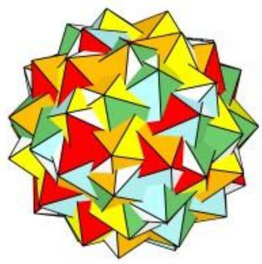
Yarim muntazam ko’pyoqlar

KEPLER-PUANSO KO’PYOQLARI

KO’PYOQLARNING AYIRIM TURLARI



Didekagrammatik antiprizma



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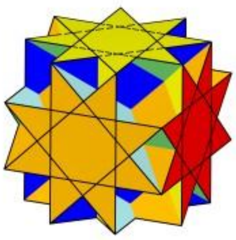

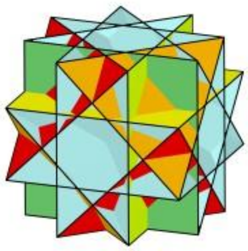

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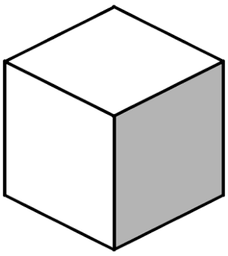
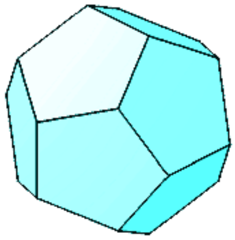
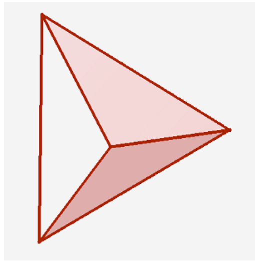
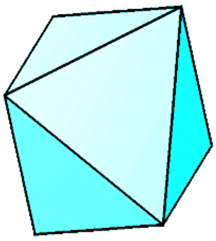
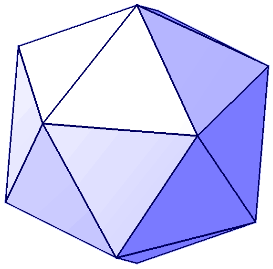
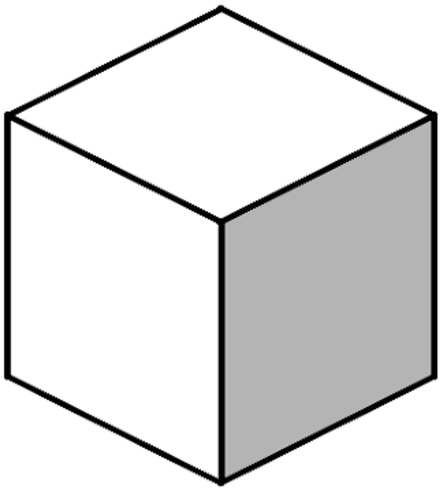






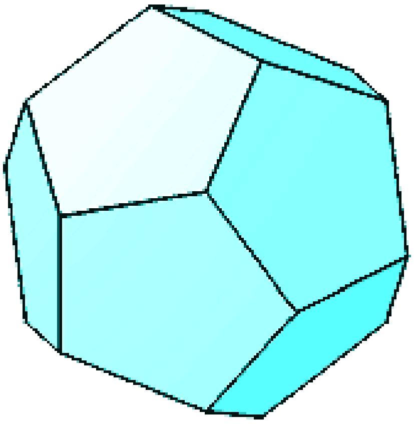
Geksaedr(KUB)

Kub oltita kvadratdan tashkil topgan. Har bir uchida uchta qirra birlashgan. Uchidagi burchaklar yigindisi 270 gradus. Kubning 6 tomoni, 8 uchi va 12 qirrasi bor.



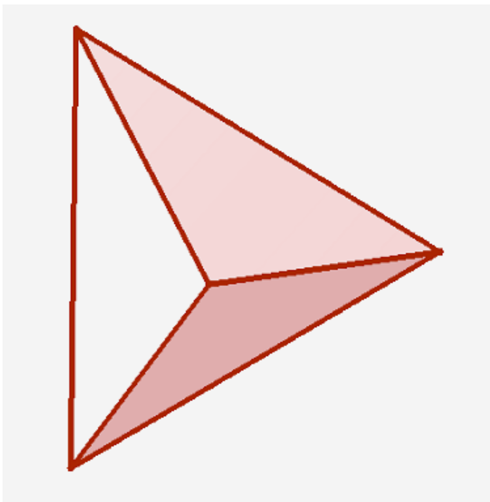
DODEKAEDR

Dodekaedr 12 ta muntazam beshburchakdan tashkil topgan.Har bir uchida uchta qirra birlashadi. Har bir uchidagi burchaklar yig'indisi 324 gradus. Dodekaedrning 12 ta tomoni, 20 ta uchi va 30 ta qirrasi bor.



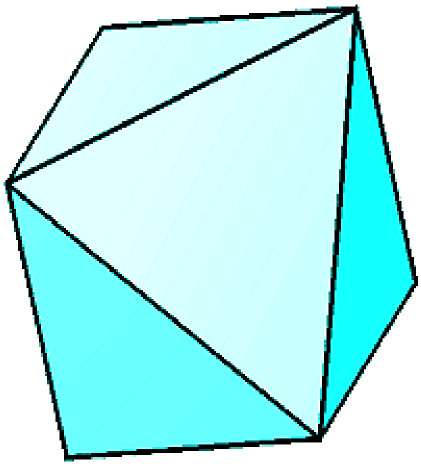
TETRAEDR

Tetraedr to'rtta muntazam uchburchakdan tashkil topgan. Har bir uchida uchta qirra birlashgan.Har bir uchidagi burchaklar yig'indisi 180 gradusga teng. Tetraedrning 4 ta tomoni, 4 ta uchi va 6 ta qirrasi bor.



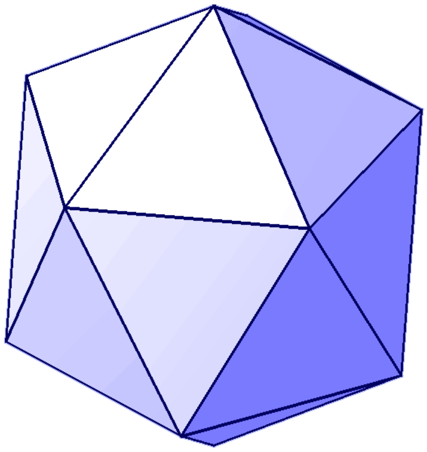
OKTAEDR

Октaedr sakkizta muntazam uchburchakdan tashkil topgan. Har bir uchida to’rtta qirra birlashgan. Har bir uchidagi burchaklar yig’indisi 240 gradus. Октaedrning 8 ta tomoni, 6 ta uchi va 12 ta qirrasi bor.

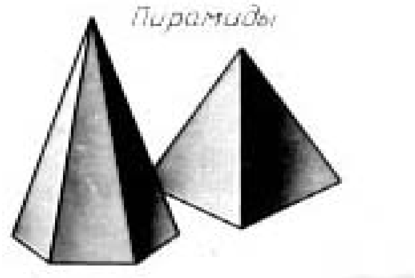
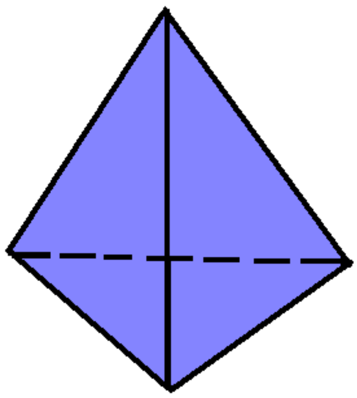


IKOSAEDR

Ikosaedr o’n ikkita muntazam uchburchakdan tashkil topgan. Har bir uchida beshta qirra birlashgan. Har bir uchidagi burchaklar yig’indisi 300 gradus. Ikosaedrning 20 ta tomoni, 12 ta uchi va 30 ta qirrasi bor.

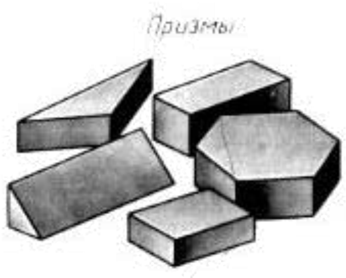


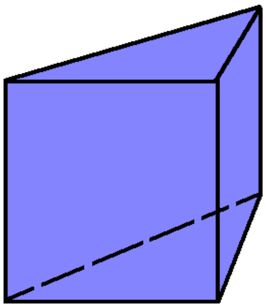
PIRAMIDA

	PIRAMIDA
Piramida – asosi ko’pburchak bo’lib, yon tomonlari uchburchaklardan iborat bo’lgan ko’pyoq. n-burchakli piramida n+1 tomonga ega. Agar piramidaning asosi muntazam ko’pburchakdan iborat bo’lsa muntazam piramida deyiladi, uning balandligi asos markaziga proeksiyalanadi.	

Prizma – Asoslari parallel ko`chish natijasida hosil bo`lgan ikkita ko`pburchak, yon tomonlari parallelogrammlardan iborat ko`pyoq. Asosi uchburchak bolgan prizma uchburchakli prizma deyiladi. Uani asosidagi ko`pburchak nomi bilan nomlanadi.

PRIZMA





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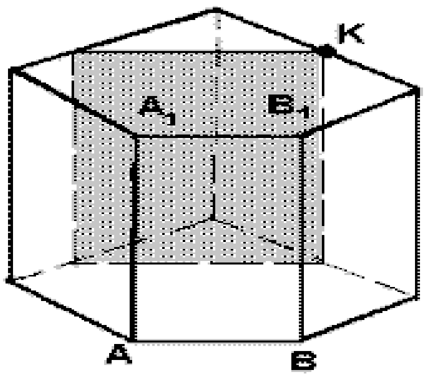

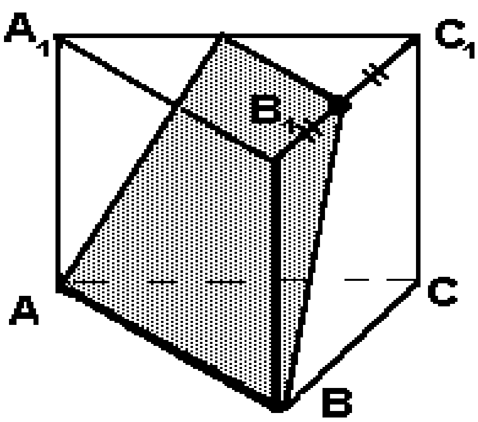

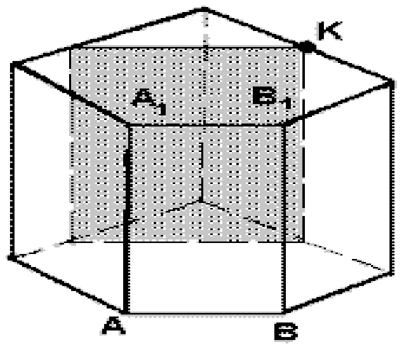

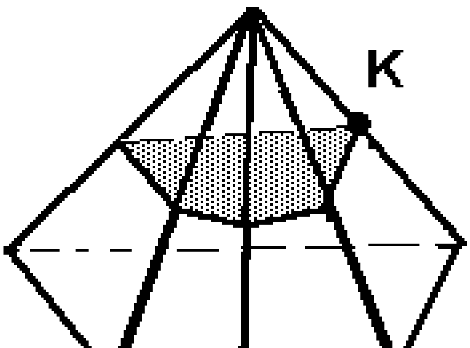

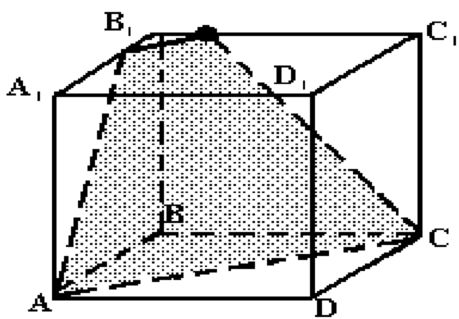

KO`PYOQLAR YOILMASI